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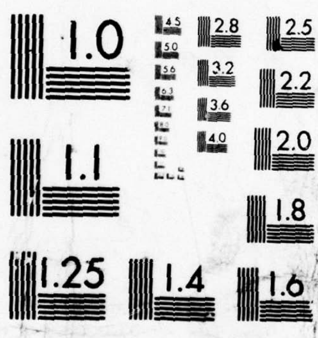

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APPROXIMATION OF DELAY SYSTEMS WITH APPLICATIONS TO CONTROL AND IDENTIFICATION

H. T. Banks

Abstract: We discuss approximation ideas for functional differential equations and how these ideas can be employed in optimal control and parameter estimation problems. Two specific schemes are described, one based on integral averages of the function being approximated, the other based on best L^2 spline approximations. An example illustrating numerical behavior of these schemes applied to an optimal control problem is presented.

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INTRODUCTION

In this presentation, we shall discuss approximation ideas that have proved useful in developing methods for solving optimal control and parameter identification problems involving delay systems. We shall focus on linear delay systems in our discussions although, as we shall later indicate, many of the ideas and results are valid for problems with nonlinear systems.

The approximation results we describe below are based on an abstract formulation due to Trotter and Kato dealing with approximations to semigroups of linear operators. In order to make use of these approximation theorems, it is necessary to reformulate our delay system as an abstract system in an appropriate Hilbert space. To this end, consider the delay system

$$\begin{aligned} \dot{x}(t) &= L(x_t) + f(t) \qquad 0 \le t \le t_1, \\ x(0) &= \eta, \quad x_0 = \phi, \end{aligned} \tag{1}$$

where, for ψ continuous on $[-r,0]$,

$$L(\psi) = \sum_{i=0}^{\nu} A_i \psi(-\tau_i) + \int_{-r}^{0} A(\theta)\psi(\theta)d\theta \tag{2}$$

with A_i, $A(\theta)$ given $n \times n$ matrices and $0 = \tau_0 < \tau_1 < \cdots < \tau_\nu \le r$. For $0 \le t \le t_1$, the function $\theta \to x_t(\theta)$, $-r \le \theta \le 0$, is given by $x_t(\theta) = x(t+\theta)$ whenever x is defined on $[-r,t_1]$.

With an appropriate interpretation of the operator L, one can define solutions x (i.e. solutions exist) to (1) corresponding to initial data (η,ϕ) in $Z \equiv R^n \times L_2([-r,0],R^n) \equiv R^n \times L_2^n(-r,0)$ and perturbations f in $L_2^n(0,t_1)$ as functions which satisfy the initial

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conditions in (1), are absolutely continuous on $(0,t_1)$ and satisfy the differential equation in (1) almost everywhere on $(0,t_1)$. One can thus define a <u>homogeneous solution semigroup</u> of operators $S(t): Z \to Z$, $t \geq 0$, by

$$S(t)(\eta,\phi) \equiv (x(t;\eta,\phi),x_t(\eta,\phi))$$

where x is the solution of (1) with $f \equiv 0$. It is not difficult to argue that this defines a strongly continuous semigroup (C_0-semigroup) $\{S(t)\}_{t\geq 0}$ of linear operators with infinitesimal generator $\mathscr{A}$ (i.e. $S(t) \sim e^{\mathscr{A}t}$) on $\mathscr{D}(\mathscr{A}) \equiv \{(\phi(0),\phi) | \phi$ is absolutely continuous with $\dot{\phi}$ in $L_2^n(-r,0)\}$ given by

$$\mathscr{A}(\phi(0),\phi) = (L(\phi),\dot{\phi}). \tag{3}$$

This semigroup can be used to give an abstract "variation of parameters" representation for solutions of (1) in Z. That is, suppose one defines for $(\eta,\phi) \in Z$ and $f \in L_2^n(0,t_1)$ the function $t \to z(t;\eta,\phi,f)$ by

$$z(t;\eta,\phi,f) = S(t)(\eta,\phi) + \int_0^t S(t-\sigma)(f(\sigma),0)d\sigma. \tag{4}$$

Then one can argue (see [2]) an equivalence between (1) and (4). More precisely, one has that

$$z(t;\eta,\phi,f) = (x(t;\eta,\phi,f),\ x_t(\eta,\phi,f))$$

for $t \geq 0$ where z is given by (4) and $t \to x(t;\eta,\phi,f)$ is the solution of (1).

For $\eta = \phi(0)$ and ϕ,f sufficiently smooth, it turns out that (4) is equivalent to an ordinary differential equation (ODE) in the Hilbert space Z which may be written

$$\begin{aligned} \dot{z}(t) &= \mathscr{A}z(t) + (f(t),0) \\ z(0) &= (\phi(0),\phi). \end{aligned} \tag{5}$$

We seek to approximate (4) (or (5)) by considering approximations in finite dimensional subspaces Z^N (where we will obtain finite dimensional ODE's). The fundamental approximation result from semigroup theory that we employ can be stated roughly as "if $\mathscr{A}^N \to \mathscr{A}$

in a proper sense, then $e^{\mathscr{A}^N t} \to e^{\mathscr{A}t}$ in some sense". One version of this result due to Trotter-Kato [13] can be stated in precise terms as follows.

<u>Theorem</u>. Suppose $\mathscr{A}^N, \mathscr{A}$ are generators of C_0-semigroups $\{S^N(t)\}$, $\{S(t)\}$ on Z satisfying:

(i) there exist M and $\beta > 0$ such that

$$|S^N(t)| \le Me^{\beta t}, \quad |S(t)| \le Me^{\beta t},$$

(ii) there exists a dense subset $\mathscr{D}$ of Z such that $\mathscr{A}^N z \to \mathscr{A}z$ as $N \to \infty$ for each $z \in \mathscr{D}$,

(iii) there exist a complex number λ_0 with real part larger than β such that $(\mathscr{A} - \lambda_0 I)\mathscr{D}$ is dense in Z.

Then $S^N(t)z \to S(t)z$ as $N \to \infty$ for every $z \in Z$, uniformly in t on compact intervals.

We wish to emphasize that use of this semigroup formulation is mainly for convenience in proving convergence of particular schemes. Just as in the case of differencing schemes for solution of partial differential equations, the semigroup formulation is not an essential aspect in the development of numerical methods. Rather, the particular choice of the subspaces Z^N and the approximating operators $\mathscr{A}^N$ is the question of utmost importance.

In both the control and identification applications discussed here, the basic system to be considered is (1) or, equivalently (4). Once we have chosen Z^N and $\mathscr{A}^N$ satisfying the Trotter-Kato hypotheses, we shall also employ (orthogonal) projections P^N of Z onto Z^N to then approximate equation (4) by

$$z^N(t) = S^N(t)P^N(\eta,\phi) + \int_0^t S^N(t-\sigma)P^N(f(\sigma),0)d\sigma. \tag{6}$$

Since in the cases of interest to us $\mathscr{A}^N$ will be bounded and $S^N(t) = e^{\mathscr{A}^N t}$, we may equivalently write this as

$$\begin{aligned} \dot{z}^N(t) &= \mathscr{A}^N z^N(t) + P^N(f(t),0) \\ z^N(0) &= P^N(\eta,\phi). \end{aligned} \tag{7}$$

The convergence of S^N to S will be sufficient to guarantee a

desired convergence of solutions of (7) in both the optimal control and parameter identification problems detailed below.

OPTIMAL CONTROL PROBLEMS

The basic idea employed in control problems is a classical one which is the basis for all Ritz type methods in approximation theory. Suppose one has an optimal control problem $(\mathcal{P})$ in a Hilbert space Z. By considering a sequence of approximating problems $(\mathcal{P}_N)$ on finite dimensional subspaces Z^N, one seeks to obtain a sequence of more easily solved problems whose solutions will approximate (i.e., approach in the limit as $N \to \infty$) solutions of the original problem $(\mathcal{P})$. To be more specific, let $f(t) = Bu(t)$ in (1) where B is an $n \times m$ matrix and u is to be chosen from some admissible class $\mathcal{U} \subset L_2^m(0,t_1)$ of control functions. The problem $(\mathcal{P})$ might typically consist of choosing $u \in \mathcal{U}$ so as to minimize (we do not distinguish between a vector and its transpose here and below)

$$\Phi(u) = x(t_1)Qx(t_1) + \int_0^{t_1} \{x(t)Wx(t) + u(t)Ru(t)\}dt, \qquad (8)$$

where x is the solution of (1) corresponding to u and the matrices Q,W and R are symmetric with Q and W positive semi-definite, R positive definite.

Writing solutions z^N of (7) in terms of components in R^n and L_2^n, $z^N = (x^N,y^N)$, and defining

$$\Phi^N(u) = x^N(t_1)Qx^N(t_1) + \int_0^{t_1}\{x^N(t)Wx^N(t) + u(t)Ru(t)\}dt, \qquad (9)$$

we take as our approximating problem $(\mathcal{P}_N)$ that of minimizing Φ^N over $\mathcal{U}$ subject to (7).

If we denote by $\bar{u},\bar{u}^N$ solutions to problems $(\mathcal{P})$ and $(\mathcal{P}_N)$ respectively, one can often use the Trotter-Kato results to guarantee convergence of the $\bar{u}^N$ to $\bar{u}$ in a desired sense. For example, in the case of Φ,Φ^N given by (8), (9) and $\mathcal{U}$ a convex closed set, one can argue that the optimal controls $\bar{u},\bar{u}^N$ exist and are unique, $\bar{u}^N \to \bar{u}$ in L_2 as $N \to \infty$, and furthermore $\Phi^N(\bar{u}^N) \to \Phi(\bar{u})$. Similar results can be obtained for more general payoff functions in place of (8) and (9). For details along with convergence arguments see [2], [3].

In many practical situations, it is important to be able to fit a model described by equations such as (1) to data obtained through empirical efforts. In this case one has certain parameters, say $q \varepsilon R^k$, on which the system depends and which must be "identified" or "estimated". For example suppose the operator L in (2) depends on parameters q (typically these may be some of the matrix coefficient terms in (2) and/or even a delay so that L might have the form $L(q;\psi) = A_0(\alpha)\psi(0) + A_1(\alpha)\psi(-\tau)$, $q = (\alpha,\tau)$). One then must identify q in the system

$$\begin{aligned} \dot{x}(t) &= L(q;x_t) + f(t) \\ c(t) &= Cx(t). \end{aligned} \tag{10}$$

Here $c(t)$ represents the "observables" for the system. One perturbs the system (via f) and collects data $\xi_1,\ldots,\xi_M$ at times $t_1,\ldots t_M$ (the ξ_i are observations for $c(t_i)$). The problem is to choose $\bar{q}$ from some admissible parameter set $\mathscr{Q}$ so that it is a <u>maximum likelihood estimator</u> (MLE) or perhaps so that it minimizes

$$J(q) = \sum_{i=1}^{M} |c(t_i;q) - \xi_i|^2. \tag{11}$$

Just as in the control problems, one reformulates this identification problem as one in the abstract space Z involving a system such as (4) (or (5)), and approximates by a sequence of identification problems. That is, one has approximating systems (again, $z^N = (x^N,y^N)$)

$$\begin{aligned} \dot{z}^N(t) &= \mathscr{A}^N(q)z^N(t) + P^N(f(t),0) \\ c^N(t) &= Cx^N(t) \end{aligned} \tag{12}$$

with observations $\{\xi_i\}$ for $\{c^N(t_i;q)\}$. One then chooses $\bar{q}^N \varepsilon \mathscr{Q} \subset R^k$ so that it is a MLE for (12) or so that it minimizes

$$J^N(q) = \sum_{i=1}^{M} |c^N(t_i;q) - \xi_i|^2, \tag{13}$$

depending on how one has decided to seek a fit to data for the original system.

Under quite reasonable assumptions, one can use the Trotter-Kato results to guarantee existence of a vector $\bar{q}$ (loosely speaking $\lim \bar{q}^N$) so that $\mathscr{A}(\bar{q})$ is the limit of $\mathscr{A}^N(\bar{q}^N)$ in an appropriate

sense and so that $\bar{q}$ is a MLE (or a minimizer for (11) if (13) is used in the approximating problems) for (10) with data $\{\xi_i\}$.

There remains the basic question of how to choose Z^N and $\mathscr{A}^N$ so that the needed Trotter-Kato hypotheses are satisfied and most importantly, so that efficient numerical schemes are generated. We shall discuss and compare two particular choices here; the first we refer to as "averaging" (AVE) approximations while the second involves spline type (SPL) approximations.

AVERAGING APPROXIMATIONS

For a given positive integer N, one partitions the interval $[-r,0]$ into N equal subintervals with a partition $\{t_j^N\}$, $t_j^N = -jr/N$, and defines the characteristic functions

$$\chi_j^N = \chi_{[t_j^N, t_{j-1}^N)}, \quad j = 2,\dots,N, \quad \chi_1^N = \chi_{[t_1^N, t_0^N]}.$$

One then takes Z^N to be the $n(N+1)$ dimensional subspace of Z defined by

$$Z^N = \{(\eta,\psi) \mid \eta \in R^n,\ \psi = \sum_{j=1}^{N} v_j^N \chi_j^N,\ v_j^N \in R^n\}.$$

The projections $P^N\colon Z \to Z^N$ are given by

$$P^N(\eta,\phi) = (\eta, \sum_1^N \phi_j^N \chi_j^N)$$

where $\phi_j^N \equiv (N/r)\int_{t_j^N}^{t_{j-1}^N} \phi(s)ds$, the integral averages of ϕ over the partition subintervals. Finally, one defines the approximating operators $\mathscr{A}^N\colon Z \to Z^N$ by

$$\mathscr{A}^N(\eta,\phi) = (L^N(\eta,\phi), \sum_{j=1}^{N} \frac{N}{r}\{\phi_{j-1}^N - \phi_j^N\}\chi_j^N)$$

where $\phi_0^N \equiv \eta$ and

$$L^N(\eta,\phi) \equiv A_0\eta + \sum_{i=1}^{\nu}\sum_{j=1}^{N} A_i\phi_j^N\chi_j^N(-\tau_i) + \sum_{j=1}^{N} A_j^N\phi_j^N$$

with $A_j^N \equiv \int_{t_j^N}^{t_{j-1}^N} A(\theta)d\theta$. With these definitions of $Z^N, \mathscr{A}^N, P^N$ one can

then argue that the desired Trotter-Kato hypotheses are satisfied and moreover that the convergence behavior necessary for use in both optimal control and identification problems is obtained (see [3],[8])

SPLINE APPROXIMATIONS

For K^{th} order spline approximations, one defines the subspace $Z^N \equiv \{(\psi^N(0),\psi^N) \mid \psi^N$ is a spline of order K on $[-r,0]$ with knots at $t_j^N = -jr/N$, $j = 0,1,\ldots,N\}$. Letting P^N be the orthogonal projection of Z onto the closed subspace Z^N, one can then define

$$\mathscr{A}^N \equiv P^N \mathscr{A} P^N$$

where $\mathscr{A}$ is given in (3).

For example, in the case of first order ("linear elements") splines, the subspace Z^N is of dimension $n(N+1)$. If one defines the usual "roof" functions for $j = 0,1,\ldots,N$ and $-r \le \theta \le 0$ by

$$e_j^N(\theta) = \begin{cases} \frac{N}{r}(\theta - t_{j+1}^N) & t_{j+1}^N \le \theta \le t_j^N \\ \frac{N}{r}(t_{j-1} - \theta), & t_j^N \le \theta \le t_{j-1}^N \\ 0 & \text{elsewhere}, \end{cases}$$

where $t_j^N = -jr/N$ for all j, then Z^N can be written

$$Z^N = \{(\psi^N(0),\psi^N) \mid \psi^N = \sum_{j=0}^{N} a_j^N e_j^N,\ a_j^N \in R^n\}.$$

Of course, $P^N(\eta,\phi) = \hat{\phi}^N = (\phi^N(0),\phi^N)$ where $\hat{\phi}^N$ is the solution to the problem of minimizing $|\hat{\zeta} - (\eta,\phi)|$ over $\hat{\zeta}$ in Z^N.

The conditions for the Trotter-Kato theorem can be verified for these spline based approximations and, in fact, the arguments, using fundamental estimates from spline analysis, are independent of the order of splines used (see [6]).

For both the AVE and SPL approximations, one can obtain estimates on the convergence rates of the approximations. The AVE method is essentially a first order method (error like 1/N) while one can argue that the error in the K^{th} order SPL scheme behaves like $1/N^K$. In actual practice our computations have revealed that the SPL methods usually converge more rapidly than these estimates predict (see the discussions in [6]) and indeed for both control and identification

problems it appears that the "first order" SPL method often offers significant advantages over the AVE method at little cost in additional complexity with respect to implementation. To partially illustrate this feature and to indicate what one might typically expect in the way of convergence for these methods, we present just one of a number of control examples to which we have applied these ideas

EXAMPLE (CONTROLLED OSCILLATOR WITH DELAYED DAMPING)

The problem is to minimize

$$\Phi(u) = 5x_1(2)^2 + \frac{1}{2}\int_0^2 u(t)^2 dt$$

over $u \in \mathscr{U} = L_2(0,2)$ subject to

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -x_1(t) - x_2(t-1) + u(t), \qquad 0 \le t \le 2, \end{aligned} \tag{14}$$

and

$$\begin{aligned} x_1(\theta) &= 10 \\ x_2(\theta) &= 0, \qquad -1 \le \theta \le 0. \end{aligned}$$

The system (14) is, of course, the vector formulation for the system dynamics $\ddot{y}(t) + \dot{y}(t-1) + y(t) = u(t)$. One can use necessary and sufficient conditions (a maximum principle for delay system problems) to solve analytically this simple example. Upon doing so one finds (see [3],[5]) the optimal control

$$\overline{u}(t) = \begin{cases} \delta \sin(2-t) + (\delta/2)(1-t)\sin(t-1) & 0 \le t \le 1 \\ \delta \sin(2-t) & 1 \le t \le 2, \end{cases}$$

where δ is the solution of a certain algebraic equation with approximate value $\delta \approx 2.5599$. The optimal value of the payoff Φ is given by

$$\overline{\Phi} = \Phi(\overline{u}) \simeq 3.399119.$$

We used a conjugate-gradient scheme to compute the solution of each of the approximate control problems $(\mathscr{P}_N)$ described above for both the AVE and SPL schemes for several values of N. Denoting the optimal

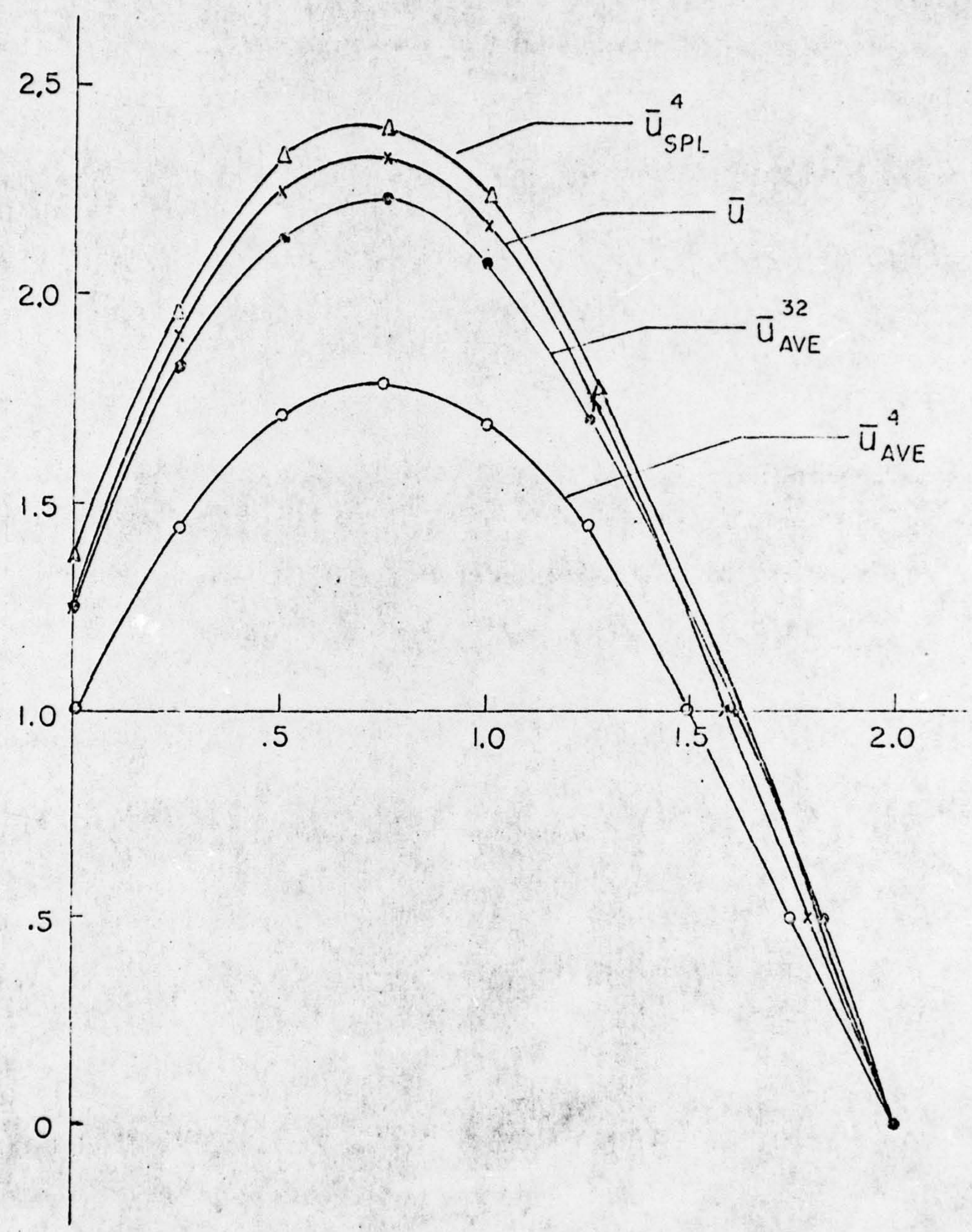

FIG. 1 OPTIMAL CONTROLS FOR OSCILLATOR WITH DELAYED DAMPING

values $\Phi^N(\bar{u}^N_{AVE})$ and $\Phi^N(\bar{u}^N_{SPL})$ of the payoffs by $\bar{\Phi}^N_{AVE}$ and $\bar{\Phi}^N_{SPL}$ respectively, we give a representative sample of our numerical findings in tabular form.

N	$\bar{\Phi}^N_{AVE}$	$\lvert\bar{\Phi}^N_{AVE}-\bar{\Phi}\rvert$	$\bar{\Phi}^N_{SPL}$	$\lvert\bar{\Phi}^N_{SPL}-\bar{\Phi}\rvert$
4	2.15154	1.2475	3.53549	.1363
8	2.67110	.7280	3.43454	.0354
16	3.00356	.3955	3.40857	.0094
32	3.19298	.2061	3.40193	.0028

We note that the convergence of $\bar{\Phi}^N_{AVE}$ to $\bar{\Phi}$ is like $1/N$ while that of $\bar{\Phi}^N_{SPL}$ is $1/N^2$. A graph comparing several of the optimal controls $\bar{u}^N_{SPL}, \bar{u}^N_{AVE}$ with the solution $\bar{u}$ of the original problem is given in Figure 1. One also has for this example that $|\bar{u}^N_{SPL}|_{L_2} \to |\bar{u}|_{L_2}$ like $1/N^2$ while $|\bar{u}^N_{AVE}|_{L_2} \to |\bar{u}|_{L_2}$ is like $1/N$.

We complete our presentation with brief mention of related results that have been obtained through efforts by our group at Brown University and our associates and colleagues.

The basic abstract framework employing the Trotter-Kato theorem to ensure convergence in optimization problems for delay systems was given in [2]. A detailed investigation of the AVE approximation and its use in linear system control problems along with several solved examples can be found in [3]. Additional examples of solved control problems along with use of the AVE scheme on these examples may be found in [5]. An extension of the theory of [2], [3] to treat control problems with nonlinear systems along with related nummerical findings are given in [1]. In efforts to find alternatives to the AVE scheme, Burns and Cliff discuss in [7] a scheme involving piecewise linear approximations combined with AVE type ideas. Spline approximations in the context of a general framework suitable for use in control and identification problems are developed in [6]. More recently, Kappel and Schappacher [10] have used interpolating splines for an approximation method that is applicable to nonlinear retarded equations while Kunisch [12] has developed the AVE scheme for nonlinear neutral functional differential equations. Kunisch's development is in the spirit of the earlier results of Kappel and Schappacher [9] who formulated a "local semigroup" approach to handle AVE approximations for locally Lipschitz nonlinear retarded equations. In [8], Burns and Cliff

discuss the use of the AVE scheme in identification problems while results for SPL based identification methods are presented in [4].

All of the above schemes, whether for AVE or SPL, lead to the approximation of a differential equation (e.g. (5)) in the Hilbert space Z by a finite-dimensional ODE. To actually use these methods in computations, one needs a second approximation (e.g. a standard Runga-Kutta or predictor-corrector scheme) to solve the approximating ODE's. A natural question that arises is "why not go directly from the original (infinite dimensional) delay system to a difference equation?" Reber, in [14], [15], investigates this question in the spirit of the functional analytic methods of the papers cited above (factor space ideas-see [11]-are employed in place of the Trotter-Kato semigroup results). Briefly, Reber considers quite general non-autonomous linear delay equations and shows that they can be formulated as an abstract system in Z

$$Tz = \Sigma(\zeta, f) \tag{15}$$

where

$$(Tz)(t) \equiv z(t) - \int_{t_0}^{t} \mathscr{A}(\sigma) z(\sigma) d\sigma$$

$$\Sigma(\zeta, f)(t) \equiv \zeta + \int_{t_0}^{t} (f(\sigma), 0) d\sigma.$$

Here ζ is the initial data (η, ϕ) and f is the perturbing function as in (1) or (5). Equation (15) is then approximated by

$$T_N z_N = \Sigma_N p_N(\zeta, f)$$

where T_N is essentially a first order difference operator and the p_N corresponds to an AVE type approximation in the state. This results in a simultaneous discretization of both "state" and "time". Reber discusses the use (advantages vs. disadvantages) of these ideas in control problems (theoretically, the analysis is slightly more complicated than that for the simple "state" discretization - i.e. ODE approximations - ideas discussed above; numerically, implementation on the computer is quite straightforward).

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17. DISTRIBUTION ST[ATEM]ENT (of th[e] abstract entered in Block 20, if different from Report)

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We discuss approximation ideas for functional differential equations and how these ideas can be employed in optimal control and parameter estimation problems. Two specific schemes are described, one based on integral averages of the function being approximated, the other based on best L_2 spline approximations.

An example illustrating numerical behavior of these schemes applied to an optimal control problem is presented.